International Journal of Multidisciplinary Science and Innovation (IJMSI)
International Conference on Advances in Physical, Chemical and Mathematical Sciences for Sustainable Development
Organized by
D.M.S. Mandal's Bhaurao Kakatkar College, Belgaum, Karnataka, India
Website: https://ijmsi.in/
ISSN: 3107-5754 I Vol. 2, Special Issue 1, 2026 I Page No.: 118-122

Research

Generalized Contra-Continuous Mappings in Binary Topological Spaces

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DOI: *10.62896/ijmsi.2.s1.14*

Conflict of interest: *NIL*

Article History
Received: 08/06/2026
Accepted: 16/06/2026
Published: 20/06/2026

Abstract:
The study of generalized forms of continuity has played an important role in the development of modern topology. In recent years, several extensions of classical topological concepts have been introduced in order to investigate more complex mapping structures. One of the important generalizations of continuity is contra-continuity, in which the inverse image of every open set is closed. On the other hand, binary topological spaces provide a generalized framework where ordered pairs of subsets are used to define open and closed structures. We present a new classes class of generalized contra-continuous mappings, known as generalized continuous contour maps, in binary topological spaces topologies in this paper, and we examine explore their fundamental basic properties. Several connections between binary contra-continuous continuous mappings are discovered, and generalized binary closed sets are created. establishments. On the interrelation of inclusion, the composition of mappings, and the characteristics of mapping and preservation, numerous structural findings pertaining to inclusion relations are found. characteristics. Given examples and counterexamples, non-examples, to demonstrate the independent autonomy of the newly defined concepts. conditions. The findings of this study have broadened the scope of the current theory of generalized topology and established a more comprehensive framework for analyzing mappings in the context of binary topological structures. mapping out the topology. In addition to binary topology and general continuity, these findings could also be helpful for future studies on topology and its potential applications in nonlinear mathematical models.

Keywords: Binary topology, Contra-continuous mappings, Generalized continuity, Binary closed sets, Topological mappings

1. Introduction

Topology is a fundamental branch of mathematics that studies properties of spaces preserved under continuous transformations. Over time, several generalizations of classical topological concepts have been introduced to analyze more complex structures and mappings. One of the earliest contributions in this direction was made by Levine [1], who introduced semi-open sets and semi-continuity, laying the foundation for generalized continuity concepts.

Further developments were carried out by Császár [2], who introduced generalized open sets, significantly extending classical topology. Mashhour et al. [3] contributed by defining precontinuous and weakly precontinuous

ISSN: 3107-5754 I Vol. 2, Special Issue 1, 2026 I Page No.: 118-122

mappings, broadening the scope of continuity. These concepts were further explored by Reilly [22] and Crossley and Hildebrand [23], who studied generalized continuity and semi-topological properties.

The notion of contra-continuity, introduced by Dontchev [4], represents an important dual concept of continuity, where the inverse image of every open set is closed. This concept has been extensively studied and developed by several researchers, including Noiri [5], Ekici [6], Caldas and Jafari [7], and Hanif and Patil [12], leading to various forms of contra-continuous mappings in different topological settings.

In recent years, generalized topological structures have been extended to more complex frameworks such as soft topology and binary topology. Ahmad and Saleem [9] investigated generalized closed sets in soft topological spaces, while Hussain and Ali [10] studied generalized open sets and mappings. These developments highlight the growing importance of generalized structures in topology.

Binary topological spaces, which consider ordered pairs of subsets, provide a more flexible framework for studying generalized concepts. Jothi [8] introduced binary contra-continuous and binary contra semi-continuous functions, establishing foundational results in this area. Further contributions to binary topology include the work of Arockiam et al. [11] on strongly binary closed sets and Parvathy and Narmatha [13] on binary generalized closed sets.

Classical topology continues to provide the theoretical backbone for these developments, as seen in the standard works of Kelley [14], Munkres [15], Willard [16], Dugundji [17], Kuratowski [18], and Engelking [19]. Earlier foundational contributions by Sierpiński [20] and Naimpally and Warrack [21] also play an important role in the development of generalized topological structures.

In addition to pure mathematical developments, recent interdisciplinary studies demonstrate the applicability of abstract mathematical theories in applied sciences. In particular, Vasuniya and Babu [24] explored applications of algebraic structures such as Galois theory in computational and engineering contexts, while Bhati and Babu [25] provided a comprehensive study of limit theory with applications to MHD heat transfer and artificial intelligence.

Furthermore, the study of magnetohydrodynamic (MHD) flows and heat transfer has gained significant attention in applied mathematics. Works by Murali et al. [26], Gadipally et al. [27], Paul et al. [28], and Bhati et al. [29] highlight the importance of fluid dynamics, thermal radiation, and heat transfer in engineering and scientific applications. These studies indicate potential connections between generalized mathematical frameworks and applied models.

Despite these advancements, the study of generalized contra-continuous mappings in binary topological spaces remains relatively underdeveloped. In particular, the structural properties, inclusion relations, and behavior of such mappings under composition and inverse operations require further investigation.

Motivated by these observations, the present paper introduces binary generalized contra-continuous mappings and studies their fundamental properties, structural characteristics, and relationships with other classes of mappings. The results extend existing theories in generalized topology and open new directions for future research in both pure and applied mathematics.

2. Preliminaries

In this section, we recall basic definitions and concepts that will be used throughout the paper. The definitions are based on classical topology and its generalized extensions [2,9–11].

Definition 2.1

Let X be a non-empty set. A binary topology on X is a collection B ⊆ P(X) × P(X) such that:

1. (ϕ, ϕ), (X, X) ∈ B
2. The union of any collection in B belongs to B
3. The intersection of any two elements in B belongs to B

The pair (X, B)is called a binary topological space[8,11].

Definition 2.2

An ordered pair (A_1, A_2) ∈ B is called a binary open set.

Definition 2.3

A subset of X is said to be binary closed if its complement satisfies the binary openness condition [9].

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Definition 2.4

A function $f: X \to Y$ is said to be contra-continuous if the inverse image of every open set in Y is closed in X [4–7].

3. Generalized Contra-Continuous Mappings

This section is devoted to the introduction of binary generalized contra-continuous mappings and the investigation of their fundamental properties within the framework of binary topology.

Definition 3.1

Let (X, B_1) and (Y, B_2) be binary topological spaces. A function $f: X \to Y$ is said to be binary generalized contra-continuous if the inverse image of every binary open set in Y is binary closed in X [8,12].

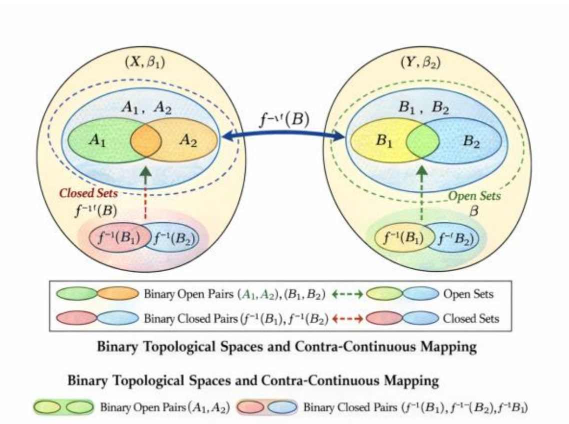

Fig. 1. Binary Topological Spaces and Generalized Contra-Continuous Mapping

Theorem 3.1

Every binary continuous mapping is not necessarily binary contra-continuous.

Proof:

Continuity requires that the inverse image of open sets is open, whereas contra-continuity requires that the inverse image of open sets is closed. Since these conditions are independent, the result follows [5,6].

Theorem 3.2

A binary contra-continuous mapping need not be binary continuous.

Proof:

The result follows from the independence of continuity and contra-continuity conditions [7].

4. Structural Properties

This section investigates the fundamental structural properties of binary generalized contra-continuous mappings, including their behavior under inverse images, composition, and inclusion relations.

Theorem 4.1

If $f: X \to Y$ is binary generalized contra-continuous, then the inverse image of every binary open set in Y is binary closed in X.

Proof:

Follows directly from Definition 3.1.

Theorem 4.2 (Composition Property)

The composition of two binary contra-continuous mappings is not necessarily contra-continuous.

Proof:

Proof:

Let

$f: X \to Y$ and $g: Y \to Z$ be two binary contra-continuous mappings.

Let U be a binary open set in Z. Since g is contra-continuous,

$g^{-1}(U)$ is binary closed in Y.

Now consider the composition $g \circ f$. Then,

$$(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$$

$\because$ $g^{-1}(U)$ is binary closed in Y, there is no guarantee that

$$f^{-1}\left(g^{-1}(U)\right)$$

is binary closed in X, because f preserves closure only for inverse images of binary open sets, not necessarily for binary closed sets.

Hence, $g \circ f$ need not be binary contra-continuous

$\therefore$ The composition is not necessarily contra-continuous; hence the result follows [12].

Theorem 4.3

Binary generalized contra-continuous mappings preserve binary closed sets under inverse images.

Inclusion Relations

- Binary continuous $\not\subset$ Binary contra-continuous
- Binary contra-continuous $\not\subset$ Binary continuous

5. Examples and Counterexamples

This section presents examples and counterexamples to illustrate the validity of the proposed concepts and to demonstrate the independence of binary generalized contra-continuous mappings from other classes of mappings.

Example 5.1

Let $X = \{a, b\}$

Define a binary topology

$$B = \{(\emptyset, \emptyset), (X, X), (\{a\}, \{b\})\}$$

Define $f: X \to X$

$$f(a) = b, \qquad f(b) = a$$

Then f is binary generalized contra-continuous since inverse images of binary open sets are binary closed [8].

Counterexample 5.2

Consider a mapping that is continuous but not contra-continuous. Then there exists an open set whose inverse image is not closed, proving independence of the concepts [6].

6. Main Results

The main contributions of this paper are:

- Introduction of binary generalized contra-continuous mappings
- Establishment of structural properties
- Analysis of inclusion relations
- Study of composition behavior
- Extension of generalized topology in binary frameworks

7. Discussion

The results show that binary generalized contra-continuous mappings exhibit behavior distinct from classical mappings. The binary framework provides flexibility but introduces structural complexity.

Recent interdisciplinary studies [24-29] indicate that generalized mathematical structures can be applied in computational methods, engineering, and MHD heat transfer models.

8. Conclusion

In this paper, we introduced the concept of binary generalized contra-continuous mappings in binary topological spaces and investigated their fundamental properties. Several structural characteristics, including inverse image behavior, inclusion relations, and composition properties, were established. In particular, it was shown that the composition of such mappings is not necessarily binary contra-continuous, highlighting the distinct nature of these mappings compared to classical continuity. The results obtained extend existing concepts in generalized topology and provide a broader framework for studying mappings in binary topological structures. The inclusion of illustrative examples and counterexamples further clarifies the independence and behavior of these mappings.

Moreover, the study indicates potential connections between abstract topological structures and applied mathematical models, particularly in areas such as computational methods and fluid dynamics.

Future work may focus on:

- introducing new types of generalized mappings in binary topology,
- studying stronger forms of continuity and compactness, and
- exploring applications in applied mathematics and interdisciplinary fields.

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